

# JEE Advanced 2024

## Sample Paper - 2

**Time Allowed: 3 hours**

**Maximum Marks: 180**

### General Instructions:

This question paper has THREE main sections and four sub-sections as below.

#### MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

#### MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

#### NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

#### MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

### Mathematics (MRQ)

1. Let  $\alpha, \beta, \gamma, \delta$  be real numbers such that  $\alpha^2 + \beta^2 + \gamma^2 \neq 0$  and  $\alpha + \gamma = 1$ . Suppose the point  $(3, 2, -1)$  is the mirror image of the point  $(1, 0, -1)$  with respect to the plane  $\alpha x + \beta y + \gamma z = \delta$ . Then which of the following statements is/are TRUE? **[4]**
  - a)  $\delta - \gamma = 3$
  - b)  $\delta + \beta = 4$
  - c)  $\delta + \beta + \gamma = \delta$
  - d)  $\alpha + \beta = 2$
2. If M and A are any two events, the probability that exactly one of them occurs is **[4]**
  - a)  $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$
  - b)  $P(M) + P(N) - P(M \cap N)$
  - c)  $P(M) + P(N) - 2P(M \cap N)$
  - d)  $P(M \cap N^c) + P(M^c \cap N)$
3. Let  $M = (a_{ij})$ ,  $i, j \in \{1, 2, 3\}$ , be the  $3 \times 3$  matrix such that  $a_{ij} = 1$ , if  $j + 1$  is divisible by  $i$ , otherwise  $a_{ij} = 0$ . Then which of the following statements is(are) true? **[4]**
  - a) The matrix  $(M - 2I)$  is invertible, where  $I$  is the  $3 \times 3$  identity matrix.
  - b)  $M$  is invertible.

c) The set  $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$

where  $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ .

d) There exists a nonzero column

matrix  $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  such that  $M \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$ .

### Mathematics (MCQ)

4. Let  $f$  be a non-negative function defined on the interval  $[0, 1]$ . If  $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$ ,  $0 \leq x \leq 1$ , and  $f(0) = 0$ , then **[3]**

a)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

b)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$

c)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

d)  $f\left(\frac{1}{2}\right) > \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) > \frac{1}{3}$

5. Let  $a > 0$ ,  $b > 0$  and  $c > 0$ . Then the roots of the equation  $ax^2 + bx + c = 0$  **[3]**

a) both are real and negative and have negative real parts

b) have negative real parts

c) have positive real parts

d) are real and negative

6. Let  $\omega$  be a complex cube root of unity with  $\omega \neq 1$ . A fair die is thrown three times. If  $r_1, r_2$  and  $r_3$  are the numbers obtained on the die, then the probability that  $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ , is **[3]**

a) 1/36

b) 1/9

c) 1/18

d) 2/9

7. A tangent to the hyperbola  $\frac{x^2}{4} - \frac{y^2}{2} = 1$  meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on: **[3]**

a)  $\frac{2}{x^2} + \frac{4}{y^2} = 1$

b)  $\frac{4}{x^2} - \frac{2}{y^2} = 1$

c)  $\frac{4}{x^2} + \frac{2}{y^2} = 1$

d)  $\frac{2}{x^2} - \frac{4}{y^2} = 1$

### Mathematics (NUM)

8. Let  $y'(x) + y(x)g'(x) = g(x)g'(x)$ ,  $y(0) = 0$ ,  $x \in \mathbb{R}$ , where  $f'(x)$  denotes  $\frac{df(x)}{dx}$  and  $g(x)$  is a given non-constant differentiable function on  $\mathbb{R}$  with  $g(0) = g(2) = 0$ . Then, the value of  $y(2)$  is **[4]**

9. Let  $z = \frac{-1 + \sqrt{3}i}{2}$ , where  $i = \sqrt{-1}$  and  $r, s \in \{1, 2, 3\}$ . Let  $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$  and I be the **[4]**

identity matrix of order 2. Then, the total number of ordered pairs  $(r, s)$  for which  $P^2 = -I$  is

10. Let X be the set of all five digit numbers formed using 1, 2, 2, 2, 4, 4, 0. For example, 22240 [4]  
is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to

11. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is equal to [4]

12. Let  $\omega = e^{\frac{i\pi}{3}}$  and a, b, c, x, y, z be non-zero complex numbers such that a + b + c = x, [4]  
 $a + b\omega + c\omega^2 = y$ ,  $a + b\omega^2 + c\omega = z$ . Then, the value of  $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$  is

13. Let k be a positive real number and let [4]

$$A = \begin{bmatrix} 2k - 1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 2k - 1 & \sqrt{k} \\ 1 - 2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If  $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$ , then [k] is equal to

### Mathematics (MATCH)

14. Let  $f_1 : R \rightarrow R$ ,  $f_2 : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow R$ ,  $f_3 : (-1, e^{\frac{\pi}{2}} - 2) \rightarrow R$  and  $f_4 : R \rightarrow R$  be functions [3]  
defined by

i.  $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$ ,

ii.  $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ , where the inverse trigonometric functions  $\tan^{-1} x$  assumes values in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ ,

iii.  $f_3(x) = [\sin(\log_e(x + 2))]$ , where, for  $t \in R$ , [t] denotes the greatest integer less than or equal to t,

iv.  $f_4(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

LIST - I	LIST - II
(P) The function $f_1$ is	(1) <b>NOT</b> continuous at $x = 0$
(Q) The function $f_2$ is	(2) continuous at $x = 0$ and <b>NOT</b> differentiable at $x = 0$
(R) The function $f_3$ is	(3) differentiable at $x = 0$ and its derivative is <b>NOT</b> continuous at $x = 0$
(S) The function $f_4$ is	(4) differentiable at $x = 0$ and its derivative is continuous at $x = 0$

a) P  $\rightarrow$  2; Q  $\rightarrow$  1; R  $\rightarrow$  4; S  $\rightarrow$  3

b) P  $\rightarrow$  4; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3

c)  $P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3$

d)  $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$

15. Match List I with List II and select the correct answer using the code given below the lists: [3]

List-I	List-II
(P) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is	(1) 100
(Q) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(2) 30
(R) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$	(3) 24
(S) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(4) 60

a)  $P \rightarrow 1, Q \rightarrow 4, R \rightarrow 3, S \rightarrow 2$

b)  $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 1, S \rightarrow 4$

c)  $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2$

d)  $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 3, S \rightarrow 1$

16. Match List I with List II and select the correct answer using the code given below the lists: [3]

List I	List II
(P) $\left( \frac{1}{y^2} \left( \frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)} \right)^2 + y^4 \right)^{\frac{1}{2}}$ take value	(1) $\frac{1}{2} \sqrt{\frac{5}{3}}$
(Q) If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos \frac{x-y}{2}$ is	(2) $\sqrt{2}$
(R) If $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right) \cos 2x$ then possible value of $\sec x$ is	(3) $\frac{1}{2}$
(S) If $\cot(\sin^{-1} \sqrt{1 - x^2}) = \sin(\tan^{-1}(x\sqrt{6}))$ , $x \neq 0$ , then possible value of $x$ is	(4) 1

a) (P) - (4), (Q) - (3), (R) - (1), (S) - (2)

b) (P) - (4), (Q) - (3), (R) - (2), (S) - (1)

c) (P) - (3), (Q) - (4), (R) - (1), (S) - (2)

d) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)

17. In a high school, a committee has to be formed from a group of 6 boys  $M_1, M_2, M_3, M_4, M_5, M_6$  and 5 girls  $G_1, G_2, G_3, G_4, G_5$ . [3]

i. Let  $\alpha_1$  be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

ii. Let  $\alpha_2$  be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

- iii. Let  $\alpha_3$  be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.
- iv. Let  $\alpha_4$  be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both  $M_1$  and  $G_1$  are **NOT** in the committee together.

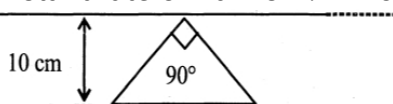
LIST-I	LIST-II
(P) The value of $\alpha_1$ is	(1) 136
(Q) The value of $\alpha_2$ is	(2) 189
(R) The value of $\alpha_3$ is	(3) 192
(S) The value of $\alpha_4$ is	(4) 200
	(5) 381
	(6) 461

- a)  $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 5; S \rightarrow 2$                       b)  $P \rightarrow 4; Q \rightarrow 6; R \rightarrow 2; S \rightarrow 1$
- c)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 3; S \rightarrow 1$                       d)  $P \rightarrow 1; Q \rightarrow 4; R \rightarrow 2; S \rightarrow 3$

### Physics (MRQ)

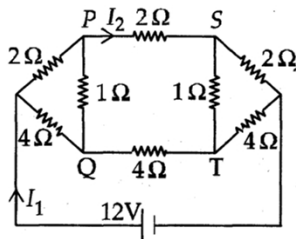
18. The filament of a light bulb has surface area  $64 \text{ mm}^2$ . The filament can be considered as a black body at temperature  $2500 \text{ K}$  emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of  $100 \text{ m}$ . Assume the pupil of the eyes of the observer to be circular with radius  $3 \text{ mm}$ . Then [4]
- (Take Stefan-Boltzmann constant =  $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ , Wien's displacement constant =  $2.90 \times 10^{-3} \text{ m-K}$ , Planck's constant =  $6.63 \times 10^{-34} \text{ Js}$ , speed of light in vacuum =  $3.00 \times 10^8 \text{ ms}^{-1}$ )
- a) radiated power entering into one eye of the observer is in the range  $3.15 \times 10^{-8} \text{ W}$  to  $3.25 \times 10^{-8} \text{ W}$                       b) the wavelength corresponding to the maximum intensity of light is  $1160 \text{ nm}$
- c) power radiated by the filament is in the range  $642 \text{ W}$  to  $645 \text{ W}$                       d) taking the average wavelength of emitted radiation to be  $1740 \text{ nm}$ , the total number of photons entering per second into one eye of the observer is in the range  $2.75 \times 10^{11}$  to  $2.85 \times 10^{11}$

19. A conducting loop in the shape of a right-angled isosceles triangle of height  $10 \text{ cm}$  is kept such that the  $90^\circ$  vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counter-clockwise direction and increased at a constant rate of  $10 \text{ A s}^{-1}$ . Which of the following statement(s) is(are) true? [4]



- a) If the loop is rotated at a constant angular speed about the wire, an additional emf of  $(\frac{\mu_0}{\pi})$  volt is induced in the wire
- b) There is a repulsive force between the wire and the loop
- c) The induced current in the wire is in opposite direction to the current along the hypotenuse
- d) The magnitude of induced emf in the wire is  $(\frac{\mu_0}{\pi})$  volt

20. For the resistance network shown in the figure, choose the correct option(s) [4]



- a)  $I_2 = 2$  A
- b) The current through PQ is zero
- c) The potential at S is less than that at Q.
- d)  $I_1 = 3$  A

### Physics (MCQ)

21. X-rays can not be diffracted by means of an ordinary grating due to: [3]

- a) Low speed
- b) large wavelength
- c) high speed
- d) short wavelength

22. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current  $I_1$  and COD carries a current  $I_2$ . The magnetic field on a point lying at a distance  $d$  from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by: [3]

- a)  $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)$
- b)  $\frac{\mu_0}{2\pi d} (I_1 + I_2)$
- c)  $\frac{\mu_0}{2\pi} \left( \frac{I_1 + I_2}{d} \right)^{\frac{1}{2}}$
- d)  $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{\frac{1}{2}}$

23. A tank is filled with water of density  $1 \text{ g per cm}^3$  and oil of density  $0.9 \text{ g per cm}^3$ . The height of water layer is  $100 \text{ cm}$  and of the oil layer is  $400 \text{ cm}$ . If  $g = 980 \text{ cm/sec}^2$ , then the velocity of efflux from an opening in the bottom of the tank is: [3]

- a)  $\sqrt{950 \times 980} \text{ cm/sec}$
- b)  $\sqrt{900 \times 980} \text{ cm/sec}$
- c)  $\sqrt{920 \times 980} \text{ cm/sec}$
- d)  $\sqrt{1000 \times 980} \text{ cm/sec}$

24. An open pipe is in resonance in 2nd harmonic with frequency  $f_1$ . Now one end of the tube is closed and frequency is increased to  $f_2$  such that the resonance again occurs in  $n$ th harmonic. Choose the correct option. [3]

a)  $n = 3, f_2 = (\frac{5}{4}) f_1$

b)  $n = 3, f_2 = (\frac{3}{4}) f_1$

c)  $n = 5, f_2 = (\frac{5}{4}) f_1$

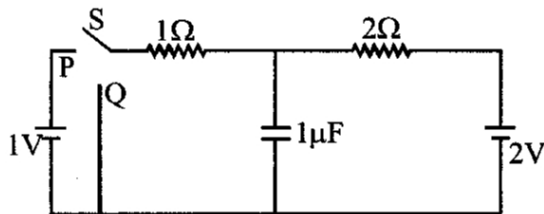
d)  $n = 5, f_2 = (\frac{3}{4}) f_1$

**Physics (NUM)**

25. A string of length 1m and mass  $2 \times 10^{-5}$ kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is \_\_\_\_\_ Newton. **[4]**

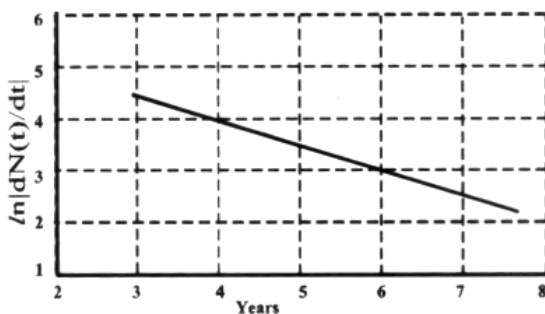
26. A projectile is fired from horizontal ground with speed v and projection angle  $\theta$ . When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is  $g' = \frac{g}{0.81}$ , then the new range is  $d' = nd$ . The value of n is \_\_\_\_\_ **[4]**

27. In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes  $q_1 \mu\text{C}$ . Then S is switched to position Q. After a long time, the charge on the capacitor is  $q_2 \mu\text{C}$ . **[4]**

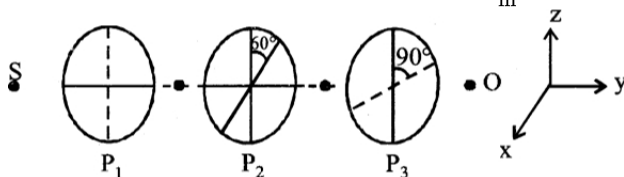


The magnitude of  $q_1$  is \_\_\_\_\_.

28. To determine the half life of a radioactive element, a student plots a graph of  $\ln \left| \frac{dN(t)}{dt} \right|$  versus t. Here  $\left| \frac{dN(t)}{dt} \right|$  is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[4]**



29. As shown in figures, three identical polaroids  $P_1, P_2$  and  $P_3$  are placed one after another. The pass axis of  $P_2$  and  $P_3$  are inclined at angle of  $60^\circ$  and  $90^\circ$  with respect to axis of  $P_1$ . The source S has an intensity of  $256 \frac{\text{W}}{\text{m}^2}$ . The intensity of light at O is \_\_\_\_\_  $\frac{\text{W}}{\text{m}^2}$ . **[4]**



30. Consider one mole of helium gas enclosed in a container at initial pressure  $P_1$  and volume  $V_1$ . It expands isothermally to volume  $4V_1$ . After this, the gas expands adiabatically and its **[4]**

volume becomes  $32V_1$ . The work done by the gas during isothermal and adiabatic expansion processes are  $W_{iso}$  and  $W_{adia}$ , respectively. If the ratio  $\frac{W_{iso}}{W_{adia}} = f \ln 2$ , then  $f$  is \_\_\_\_\_.

### Physics (MATCH)

31. A planet of mass  $M$ , has two natural satellites with masses  $m_1$  and  $m_2$ . The radii of their circular orbits are  $R_1$  and  $R_2$  respectively, ignore the gravitational force between the satellites. Define  $v_1, L_1, K_1$  and  $T_1$  to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and  $v_2, L_2, K_2$ , and  $T_2$  to be the corresponding quantities of satellite 2. Given  $\frac{m_1}{m_2} = 2$  and  $\frac{R_1}{R_2} = \frac{1}{4}$ , match the ratios in List - I to the numbers in List - II. [3]

LIST - I	LIST - II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

- a)  $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4$                       b)  $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3$   
 c)  $P \rightarrow 3; Q \rightarrow 2; R \rightarrow 4; S \rightarrow 1$                       d)  $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 4; S \rightarrow 1$
32. A series LCR circuit is connected to a  $45 \sin(\omega t)$  Volt source. The resonant angular frequency of the circuit is  $10^5 \text{ rad s}^{-1}$  and current amplitude at resonance is  $I_0$ . When the angular frequency of the source is  $\omega = 8 \times 10^4 \text{ rad s}^{-1}$ , the current amplitude in the circuit is  $0.05 I_0$ . If  $L = 50 \text{ mH}$ , match each entry in List-I with an appropriate value from List-II and choose the correct option. [3]

List-I	List-II
(P) $I_0$ in mA	(1) 44.4
(Q) The quality factor of the circuit	(2) 18
(R) The bandwidth of the circuit in $\text{rad s}^{-1}$	(3) 400
(S) The peak power dissipated at resonance in Watt	(4) 2250
	(5) 500

- a)  $P \rightarrow 4, Q \rightarrow 5, R \rightarrow 3, S \rightarrow 1$                       b)  $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 5, S \rightarrow 1$   
 c)  $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$                       d)  $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 5$
33. Match the temperature of a black body given in List I with an appropriate statement in List II, and choose the correct option. [3]

[Given: Wien's constant as  $2.9 \times 10^{-3} \text{ m-K}$  and  $\frac{hc}{e} = 1.24 \times 10^{-6} \text{ V-m}$ ]



List - I	List - II
(P) 2000 K	(1) The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4eV.
(Q) 3000 K	(2) The radiation at peak wavelength is visible to human eye.
(R) 5000 K	(3) The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S) 10000 K	(4) The power emitted per unit area is $\frac{1}{16}$ of that emitted by a blackbody at temperature 6000 K.
	(5) The radiation at peak emission wavelength can be used to image human bones.

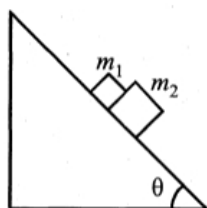
a)  $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 1$

b)  $P \rightarrow 1, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 3$

c)  $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 4, S \rightarrow 1$

d)  $P \rightarrow 3, Q \rightarrow 5, R \rightarrow 2, S \rightarrow 3$

34. A block of mass  $m_1 = 1$  kg another mass  $m_2 = 2$  kg, are placed together (see figure) on an inclined plane with angle of inclination  $\theta$ . Various values of  $\theta$  are given in List-I. The coefficient of friction between the block  $m_1$  and plane is always zero. The coefficient of static and dynamic friction between the block  $m_2$  and the plane are equal to  $\mu = 0.3$ . In List-II expressions for the friction on block  $m_2$  are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by  $g$ . [Useful information:  $\tan(5.5^\circ) \approx 0.1; \tan(11.5^\circ) \approx 0.2; \tan(16.5^\circ) \approx 0.3$ ]



List - I	List - II
(P) $\theta = 5^\circ$	(i) $m_2 g \sin \theta$
(Q) $\theta = 10^\circ$	(ii) $(m_1 + m_2) g \sin \theta$
(R) $\theta = 15^\circ$	(iii) $\mu m_2 g \cos \theta$
(S) $\theta = 20^\circ$	(iv) $\mu (m_1 + m_2) g \cos \theta$

a) (P) - (ii), (Q) - (ii), (R) - (ii), (S) - (iii)

b) (P) - (i), (Q) - (i), (R) - (i), (S) - (iii)

c) (P) - (i), (Q) - (ii), (R) - (ii), (S) - (iii)

d) (P) - (ii), (Q) - (ii), (R) - (ii), (S) - (iv)

### Chemistry (MRQ)

35. A positive carbylamine test is given by

[4]

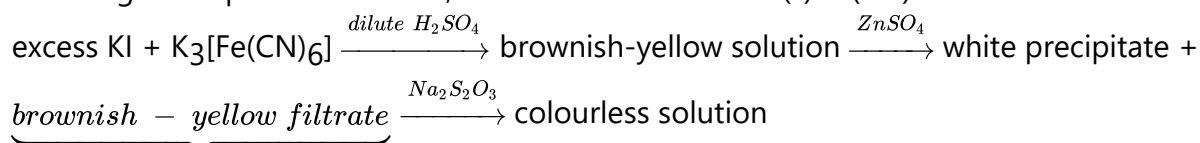
a) N-methyl-o-methylaniline

b) 2, 4-dimethylaniline

c) p-methylbenzylamine

d) N, N-dimethylaniline

36. For the given aqueous reactions, which of the statement (s) is (are) true? [4]



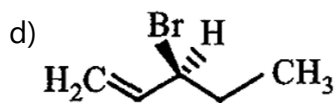
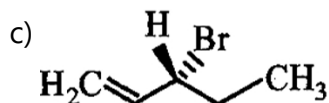
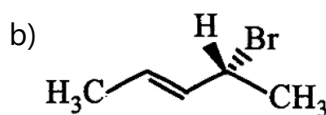
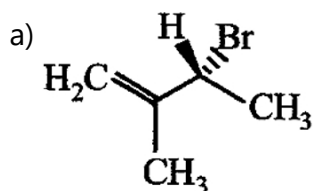
a) The first reaction is a redox reaction.

b) White precipitate is  $Zn_3[Fe(CN)_6]_2$ .

c) Addition of filtrate to starch solution gives blue colour.

d) White precipitate is soluble in NaOH solution.

37. Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is (are) [4]



### Chemistry (MCQ)

38. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens? [3]

a) Nitric acid

b) Hydrochloric acid

c) Sodium hydroxide

d) Ammonia

39. The standard reduction potential values of three metallic cations, X, Y, Z are 0.52, - 3.03 and - 1.18 V respectively. The order of reducing power of the corresponding metals is [3]

a)  $Z > Y > X$

b)  $Z > X > Y$

c)  $Y > Z > X$

d)  $X > Y > Z$

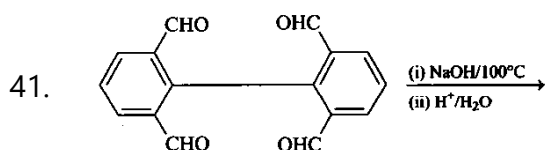
40. Which one of the following is the strongest base? [3]

a)  $AsH_3$

b)  $NH_3$

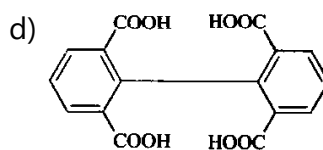
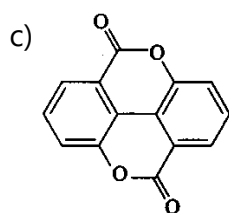
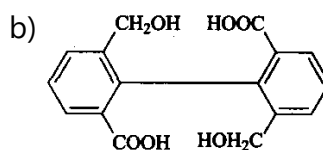
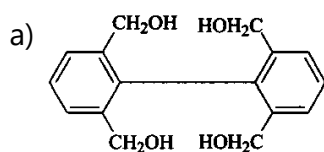
c)  $PH_3$

d)  $SbH_3$



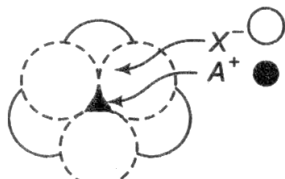
Major product is:

[3]



### Chemistry (NUM)

42. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm<sup>2</sup>. The conductance of this solution was found to be  $5 \times 10^{-7}$  S. The pH of the solution is 4. The value of limiting molar conductivity ( $\Lambda_m^\circ$ ) of this weak monobasic acid in an aqueous solution is  $Z \times 10^2$  S cm<sup>2</sup> mol<sup>-1</sup>. The value of Z is [4]
43. The arrangement of X<sup>-</sup> ions around A<sup>+</sup> ion in solid AX is given in the figure (not drawn to scale). If the radius of X<sup>-</sup> is 250 pm, the radius of A<sup>+</sup> is [4]

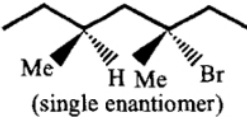


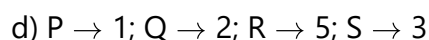
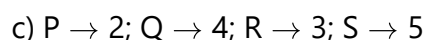
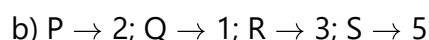
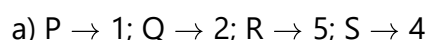
44. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is KCN, K<sub>2</sub>SO<sub>4</sub>, (NH<sub>4</sub>)<sub>2</sub>C<sub>2</sub>O<sub>4</sub>, NaCl, Zn(NO<sub>3</sub>)<sub>2</sub>, FeCl<sub>3</sub>, K<sub>2</sub>CO<sub>3</sub>, NH<sub>4</sub>NO<sub>3</sub> and LiCN [4]
45. A closed vessel with rigid walls contains 1 mole of <sup>238</sup><sub>92</sub>U and 1 mole of air at 298 K. Considering complete decay of <sup>238</sup><sub>92</sub>U to <sup>206</sup><sub>82</sub>Pb, the ratio of the final pressure to the initial pressure of the system at 298 K is [4]
46. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas Q. The amount of CuSO<sub>4</sub> (in g) required to completely consume the gas Q is [4]  
 \_\_\_\_\_  
 [Given: Atomic mass of H = 1, O = 16, Na = 23, P = 31, S = 32, Cu = 63]
47. Among the complex ions, [Co(NH<sub>2</sub>CH<sub>2</sub>CH<sub>2</sub>-NH<sub>2</sub>)<sub>2</sub>Cl]<sup>+</sup>, [CrCl<sub>2</sub>(C<sub>2</sub>O<sub>4</sub>)<sub>2</sub>]<sup>3-</sup>, [Fe(H<sub>2</sub>O)<sub>4</sub>(OH)<sub>2</sub>]<sup>+</sup>, [Fe(NH<sub>3</sub>)<sub>2</sub>(CN)<sub>4</sub>]<sup>-</sup>, Co(NH<sub>2</sub>-CH<sub>2</sub>-CH<sub>2</sub>-NH<sub>2</sub>)<sub>2</sub>(NH<sub>3</sub>)Cl]<sup>2+</sup> and [Co(NH<sub>3</sub>)<sub>4</sub>(H<sub>2</sub>O)Cl]<sup>2+</sup> the number of complex ion(s) that show(s) cis-trans isomerism is [4]

### Chemistry (MATCH)

48. Match the reactions in List-I with the features of their products in List-II and choose the correct option. [3]

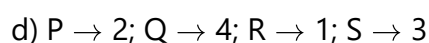
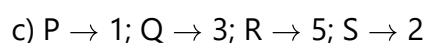
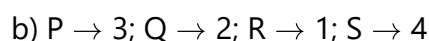
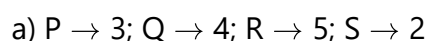
List-I	List-II
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(P) (-) - 1 - Bromo - 2 - ethylpentane (single enantiomer) $\xrightarrow[\text{S}_{\text{N}}2 \text{ reaction}]{\text{aq. NaOH}}$	(1) Inversion of configuration
(Q) (-) - 2 - Bromopentane (single enantiomer) $\xrightarrow[\text{S}_{\text{N}}2 \text{ reaction}]{\text{aq. NaOH}}$	(2) Retention of configuration
(R) (-) - 3 - Bromo - 3 - methylhexane (single enantiomer) $\xrightarrow[\text{S}_{\text{N}}1 \text{ reaction}]{\text{aq. NaOH}}$	(3) Mixture of enantiomers
(S)  (single enantiomer) $\xrightarrow[\text{S}_{\text{N}}1 \text{ reaction}]{\text{aq. NaOH}}$	(4) Mixture of structural isomers
	(5) Mixture of diastereomers



49. The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option. [3]

List-I	List-II
(P) Etard reaction	(1) Acetophenone $\xrightarrow{\text{Zn-Hg, HCl}}$
(Q) Gattermann reaction	(2) Toluene $\xrightarrow[\text{(ii) SOCl}_2]{\text{(i) KMnO}_4, \text{KOH}, \Delta}$
(R) Gattermann-Koch reaction	(3) Benzene $\xrightarrow[\text{anhyd. AlCl}_3]{\text{CH}_3\text{Cl}}$
(S) Rosenmund reduction	(4) Aniline $\xrightarrow[273-278\text{K}]{\text{NaNO}_2/\text{HCl}}$
	(5) Phenol $\xrightarrow{\text{Zn}, \Delta}$



50. Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on  $[\text{H}^+]$  are given in LIST-II. (Note: Degree of dissociation ( $\alpha$ ) of weak acid and weak base is  $\ll 1$ ; degree of hydrolysis of salt  $\ll 1$ ;  $[\text{H}^+]$  represents the concentration of  $\text{H}^+$  ions) [3]

LIST-I	LIST-II
(P) (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	(1) the value of $[\text{H}^+]$ does not change on dilution
(Q) (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	(2) the value of $[\text{H}^+]$ changes to half of its initial value on dilution



(R) (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	(3) the value of $[H^+]$ changes to two times of its initial value on dilution
(S) 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	(4) the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	(5) the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

a) P - 4; Q - 2; R - 3; S - 1

b) P - 1; Q - 5; R - 4; S - 1

c) P - 4; Q - 3; R - 2; S - 3

d) P - 1; Q - 4; R - 5; S - 3

51. Consider the Bohr's model of a one - electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the  $n^{\text{th}}$  orbit of the atom and List-II contains options showing how they depend on n [3]

List-I	List-II
(I) Radius of the $n^{\text{th}}$ orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the $n^{\text{th}}$ orbit of the atom	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the $n^{\text{th}}$ orbit	(R) $\propto n^0$
(IV) Potential energy of the electron in the $n^{\text{th}}$ orbit	(S) $\propto n^1$
	(T) $\propto n^2$
	(U) $\propto n^{\frac{1}{2}}$

Which of the following options has the correct Combination considering List-I and List-II?

a) (I), (P)

b) (I), (T)

c) (II), (Q)

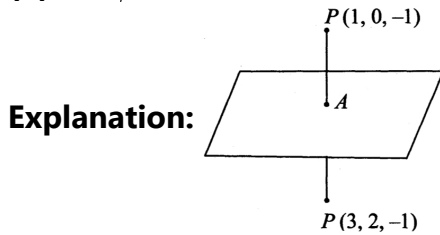
d) (II), (R)



**JEE Advanced 2024**  
**Sample Paper - 2**  
**Solution**

**Mathematics (MRQ)**

1. (a)  $\delta - \gamma = 3$   
(b)  $\delta + \beta = 4$   
(d)  $\alpha + \beta = 2$



Mid-point of PQ = A (2, 1, -1)

D.r's of PQ = 2, 2, 0

Since PQ perpendicular to plane and mid-point lies on plane

$\therefore$  Equation of plane :  $2(x - 2) + 2(y - 1) + 0(z + 1) = 0 \Rightarrow x - 2 + y - 1 = 0$

$\Rightarrow x + y = 3$  comparing with  $ax + \beta y + \gamma z = \delta$ ,

we get  $\alpha = 1, \beta = 1, \gamma = 0$  and  $\delta = 3$ .

$\therefore$  option (a), (b), (c) are true,

2. (a)  $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$   
(c)  $P(M) + P(N) - 2P(M \cap N)$   
(d)  $P(M \cap N^C) + P(M^C \cap N)$

**Explanation:**

- $P(M) + P(N) - 2P(M \cap N)$   
=  $P(M) - P(M \cap N) + P(N) - P(M \cap N)$   
=  $P(M \cap N^C) + P(M^C \cap N)$   
 $\Rightarrow$  Prob. that exactly one of M and N occurs.
- $P(M) + P(N) - P(M \cap N) = P(M \cup N)$   
 $\Rightarrow$  Prob. that at least one of M and N occurs.
- $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$   
=  $1 - P(M) + 1 - P(N) - 2[1 - P(M \cup N)]$   
=  $P(M) + P(N) - 2P(M \cup N)$   
=  $P(M) - P(M \cap N) + P(N) - P(M \cap N)$   
=  $P(M \cap N^C) + P(M^C \cap N)$   
 $\Rightarrow$  Prob. that exactly one of M and N occurs.
- $P(M \cap N^C) + P(M^C \cap N)$   
 $\Rightarrow$  Prob that M occurs but not N or prob that M does not occur but N occurs.  
 $\Rightarrow$  Prob. that exactly one of M and N occurs.

Thus we can conclude that  $P(M) + P(N) - 2P(M \cap N)$ ,  $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$  and  $P(M \cap N^C) + P(M^C \cap N)$  are the correct options.



3. (c) The set  $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$  where  $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ .

(d) There exists a nonzero column matrix  $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  such that  $M \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$ .

**Explanation:** Given that  $a_{ij} = -1$  if  $j + 1$  is divisible by  $i$ , otherwise  $a_{ij} = 0$

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

i.  $|M| = -1 + 1 = 0 \Rightarrow M$  is singular so non-invertible.

So,  $M$  is invertible is incorrect.

$$\text{ii. } M \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$$

$$\left. \begin{array}{l} a_1 + a_2 + a_3 = -a_1 \\ a_1 + a_3 = -a_2 \\ a_2 = -a_3 \end{array} \right\} \Rightarrow a_1 = 0 \text{ and } a_2 + a_3 = 0 \text{ infinite solutions.}$$

So, There exists a nonzero column matrix  $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  such that  $M \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$  is correct.

$$\text{iii. } MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + y + z = 0$$

$$x + z = 0$$

$$y = 0$$

$\therefore$  Infinite solution.

So, The set  $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$  where  $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  is correct.

$$\text{iv. } M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$|M - 2I| = 0$$

So, The matrix  $(M - 2I)$  is invertible, where  $I$  is the  $3 \times 3$  identity matrix is incorrect.

### Mathematics (MCQ)

4. (a)  $f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

**Explanation:** Given that  $f$  is a non negative function defined on  $[0, 1]$  and

$$\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1$$

Differentiating both sides with respect to  $x$ , we get  $\sqrt{1 - [f'(x)]^2} = f(x)$

$$\Rightarrow 1 - [f'(x)]^2 = [f(x)]^2 \Rightarrow [f'(x)]^2 = 1 - [f(x)]^2$$



$$\Rightarrow \frac{d}{dx} f(x) = \pm \sqrt{1 - [f(x)]^2} \Rightarrow \pm \frac{df(x)}{\sqrt{1 - [f(x)]^2}} = dx$$

Integrating both sides with respect to x, we get

$$\pm \int \frac{df(x)}{\sqrt{1 - [f(x)]^2}} = \int dx \Rightarrow \pm \sin^{-1} f(x) = x + C$$

$\therefore$  Given that  $f(0) = 0 \Rightarrow C = 0$

Hence  $f(x) = \pm \sin x$

But as  $f(x)$  is a non negative function on  $[0, 1]$

$\therefore f(x) = \sin x$

Now  $\sin x < x, \forall x > 0$

$\therefore f\left(\frac{1}{2}\right) < \frac{1}{2}$  and  $f\left(\frac{1}{3}\right) < \frac{1}{3}$

5. (a) both are real and negative and have negative real parts

**Explanation:** Since,  $a, b, c > 0$ ; therefore  $a, b, c$  should be real because order relation is not defined in the set of complex numbers.

$\therefore$  Roots of equation are either real or complex conjugate.

Let  $\alpha, \beta$  be the roots of  $ax^2 + bx + c = 0$ , then

$$\alpha + \beta = -\frac{b}{a} = -ve, \alpha\beta = \frac{c}{a} = +ve$$

$\Rightarrow$  Either both  $\alpha, \beta$  are -ve, if roots are real or both  $\alpha, \beta$  have -ve real parts, if roots are complex conjugate.

6.

(d) 2/9

**Explanation: Sample space:** A dice is thrown thrice,  $n(s) = 6 \times 6 \times 6$

**Favorable events:**  $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

i.e.  $(r_1, r_2, r_3)$  are ordered 3 triples which can take values,

$(1, 2, 3), (1, 5, 3), (4, 2, 3), (4, 5, 3)$   
 $(1, 2, 6), (1, 5, 6), (4, 2, 6), (4, 5, 6)$  } i.e. 8 ordered pairs and each can be arranged in 3!

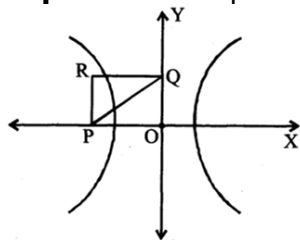
ways = 6

$$\therefore n(E) = 8 \times 6 \Rightarrow P(E) = \frac{8 \times 6}{6 \times 6 \times 6} = \frac{2}{9}$$

7.

$$(b) \frac{4}{x^2} - \frac{2}{y^2} = 1$$

**Explanation:** Equation of the tangent at the point  $\theta$  is



$$\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$$

$$\Rightarrow P = (a \cos \theta, 0) \text{ and } Q = (0, -b \cot \theta)$$

Let R be  $(h, k)$

$$\Rightarrow h = a \cos \theta, k = -b \cot \theta$$

$$\Rightarrow \frac{k}{h} = \frac{-b}{a \sin \theta} \Rightarrow \sin \theta = \frac{-bh}{ak} \text{ and } \cos \theta = \frac{h}{a}$$

By squaring and adding,  $\frac{b^2 h^2}{a^2 k^2} + \frac{h^2}{a^2} = 1$

$$\Rightarrow \frac{b^2}{k^2} + 1 = \frac{a^2}{h^2} \Rightarrow \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$$

Now, given eq<sup>n</sup> of hyperbola is  $\frac{x^2}{4} - \frac{y^2}{2} = 1$



$$\Rightarrow a^2 = 4, b^2 = 2$$

$$\therefore R \text{ lies on } \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1 \text{ i.e. } \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

### Mathematics (NUM)

8.0

Explanation:

$$\frac{dy}{dx} + y \cdot g'(x) = g(x) g'(x)$$

$$IF = e^{\int g'(x) dx} = e^{g(x)}$$

$$\therefore \text{Solution is } y(e^{g(x)}) = \int g(x) \cdot g'(x) \cdot e^{g(x)} dx + C$$

$$\text{Put } g(x) = t, g'(x) dx = dt$$

$$y(e^{g(x)}) = \int t \cdot e^t dt + C$$

$$= t \cdot e^t - \int 1 \cdot e^t dt + C = t \cdot e^t - e^t + C$$

$$y e^{g(x)} = (g(x) - 1) e^{g(x)} + C \dots(i)$$

$$\text{Given, } y(0) = 0, g(0) = g(2) = 0$$

$\therefore$  Eq. (i) becomes,

$$y(0) \cdot e^{g(0)} = (g(0) - 1) \cdot e^{g(0)} + C$$

$$\Rightarrow 0 = (-1) \cdot 1 + C \Rightarrow C = 1$$

$$\therefore y(x) \cdot e^{g(x)} = (g(x) - 1) e^{g(x)} + 1$$

$$\Rightarrow y(2) \cdot e^{g(2)} = (g(2) - 1) e^{g(2)} + 1, \text{ where } g(2) = 0$$

$$\Rightarrow y(2) \cdot 1 = (-1) \cdot 1 + 1$$

$$y(2) = 0$$

9.1

Explanation:

$$\text{Here, } z = \frac{-1+i\sqrt{3}}{2} = \omega$$

$$\therefore P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} \omega^{2r} + \omega^{4s} & \omega^{r+2s} [(-1)^r + 1] \\ \omega^{r+2s} [(-1)^r + 1] & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

$$\text{Given, } P^2 = -I$$

$$\omega^{2r} + \omega^{4s} = -1 \text{ and } \omega^{r+2s} [(-1)^r + 1] = 0$$

$$\text{Since, } r \in \{1, 2, 3\} \text{ and } (-1)^r + 1 = 0$$

$$\Rightarrow r = \{1, 3\}$$

$$\text{Also, } \omega^{2r} + \omega^{4s} = -1$$

$$\text{If } r = 1, \text{ then } \omega^2 + \omega^{4s} = -1$$

which is only possible, when  $s = 1$ .

$$\text{As, } \omega^2 + \omega^4 = -1$$

$$\therefore r = 1, s = 1$$

Again, if  $r = 3$ , then

$$\omega^6 + \omega^{4s} = -1$$

$$\Rightarrow \omega^{4s} = -2 \text{ [never possible]}$$

$$\therefore r \neq 3$$

$\Rightarrow (r, s) = (1, 1)$  is the only solution.

Hence, the total number of ordered pairs is 1.

10. 31.0

Explanation:

Number of five digit numbers divisible by 5

$$\left. \begin{array}{l} \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|4}{|3} = 4 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|4}{|2} = 12 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|4}{|3} = 4 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|4}{|2|2} = 6 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|4}{|2} = 12 \end{array} \right\} \text{Total} = 38$$

Number of five digit numbers divisible by 5 but not by 20

$$\left. \begin{array}{l} \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|3}{|3} = 1 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|3}{|2} = 3 \\ \underbrace{\quad\quad\quad}_0 \text{ fixed} \rightarrow \frac{|3}{|2} = 3 \end{array} \right\} \text{Total} = 7$$

$$\therefore p = \frac{38-7}{38} \therefore 38p = 31$$

11. 9

Explanation:

$$\text{Then, } f'(x) = \lambda(x - \alpha)(x - \beta)$$

$$\text{Here, } p'(x) = \lambda(x - 1)(x - 3) = \lambda(x^2 - 4x + 3)$$

On integrating both sides between 1 to 3, we get

$$\int_1^3 p'(x) dx = \int_1^3 \lambda(x^2 - 4x + 3) dx$$

$$\Rightarrow (p(x))_1^3 = \lambda \left( \frac{x^3}{3} - 2x^2 + 3x \right)_1^3$$

$$\Rightarrow p(3) - p(1) = \lambda \left( (9 - 18 + 9) - \left( \frac{1}{3} - 2 + 3 \right) \right)$$

$$\Rightarrow 2 - 6 = \lambda \left\{ \frac{-4}{3} \right\}$$

$$\Rightarrow \lambda = 3$$

$$\Rightarrow p'(x) = 3(x - 1)(x - 3)$$

$$\therefore p'(0) = 9$$

12. 3

Explanation:

$$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{x\bar{x} + y\bar{y} + z\bar{z}}{|a|^2 + |b|^2 + |c|^2}$$

$$(a + b + c)(\bar{a} + \bar{b} + \bar{c}) + (a + b\omega + c\omega^2)$$

$$= \frac{(\bar{a} + \bar{b}\omega^2 + \bar{c}\omega) + (a + b\omega^2 + c\omega)(\bar{a} + \bar{b}\omega + \bar{c}\omega^2)}{|a|^2 + |b|^2 + |c|^2}$$

$$= \frac{3(|a|^2 + |b|^2 + |c|^2)}{|a|^2 + |b|^2 + |c|^2} = 3$$

13. 4

Explanation:

$$|A| = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix} [C_2 \rightarrow C_2 - C_3]$$

$$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix} [R_2 \rightarrow R_2 - R_3]$$

$$= (1+2k)(8k-4k+4k^2+1) = (2k+1)^3$$

Since B is skew symmetric of odd order,

$$\therefore |B| = 0$$

$$\text{Hence, } |\text{Adj } A| + |\text{Adj } B| = |A|^2 + |B|^2 = 10^6$$

$$\Rightarrow (2k+1)^6 = 10^6 \Rightarrow k = 4.5, \therefore [k] = 4$$

**Mathematics (MATCH)**

14. (a) P  $\rightarrow$  2; Q  $\rightarrow$  1; R  $\rightarrow$  4; S  $\rightarrow$  3

**Explanation:**

$$\text{i. } f'_1(0) = \lim_{h \rightarrow 0} \left[ \frac{\sin \sqrt{1-e^{-h^2}} - 0}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{\sin \sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \times \frac{\sin \sqrt{1-e^{-h^2}}}{h^2} \times \frac{|h|}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ 1 \times 1 \times \frac{|h|}{h} \right] = \lim_{h \rightarrow 0} \frac{|h|}{h} \left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

which does not exist.

$\therefore$  for (P), (2) is correct.

$$\text{ii. } \lim_{x \rightarrow 0} f_2(x) = \lim_{x \rightarrow 0} \left[ \frac{|\sin x|}{\tan^{-1} x} \right]$$

$$\lim_{x \rightarrow 0} \left[ \frac{|\sin x|}{|x|} \times \frac{x}{\tan^{-1} x} \times \frac{|x|}{x} \right]$$

$$\lim_{x \rightarrow 0} \left[ 1 \times 1 \times \frac{|x|}{x} \right] = \lim_{x \rightarrow 0} \frac{|x|}{x} \left[ \because \lim_{x \rightarrow \infty} \frac{x}{\tan^{-1} x} = 1 \right]$$

which does not exist, so for Q, (1) is correct.

$$\text{iii. } \lim_{x \rightarrow 0} f_3(x) = \lim_{x \rightarrow 0} [\sin(\log_e(x+2))]$$

$$\text{if } x \rightarrow 0 \Rightarrow (x+2) \rightarrow 2 \Rightarrow \log_e(x+2) \rightarrow \log_e 2 < 1$$

$$\Rightarrow 0 < \lim_{x \rightarrow 0} \sin(\log_e(x+2)) < \sin 1$$

$$\Rightarrow \lim_{x \rightarrow 0} [\sin(\log_e(x+2))] = 0$$

$$f_3(x) = 0 \quad \forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$$

$$\Rightarrow f'_3(x) = 0 \quad \forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$$

$$\Rightarrow f''_3(x) = 0 \quad \forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$$

$\therefore$  for (R), (4) is correct.

$$\text{iv. } \lim_{x \rightarrow 0} f_4(x) = \lim_{x \rightarrow 0} \left( x^2 \sin \frac{1}{x} \right) = \lim_{x \rightarrow 0} x^2 \left( \sin \frac{1}{x} \right) = 0$$

$$f'_4(0) = \lim_{x \rightarrow 0} \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h} = \lim_{x \rightarrow 0} h \sin\left(\frac{1}{h}\right) = 0$$

$$f'_4(x) = -\cos \frac{1}{x} + 2x \sin \frac{1}{x}, x \neq 0$$

$$\lim_{x \rightarrow 0} f'_4(x) = \lim_{x \rightarrow 0} \left[ -\cos \frac{1}{x} + 2x \sin \frac{1}{x} \right] = -\lim_{x \rightarrow 0} \cos \frac{1}{x}$$

which does not exist

So for (S), (3) is correct.

15.

(c)  $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2$

**Explanation:**

P. Given that  $[\vec{a} \ \vec{b} \ \vec{c}] = 2$

$$\begin{aligned} \therefore & [2(\vec{a} \times \vec{b}) 3(\vec{b} \times \vec{c}) \vec{c} \times \vec{a}] \\ &= 6 [\vec{a} \times \vec{b} \ \vec{b} \times \vec{c} \ \vec{c} \times \vec{a}] \\ &= 6 [\vec{a} \ \vec{b} \ \vec{c}]^2 = 6 \times 4 = 24 \\ \therefore & (P) \rightarrow (3) \end{aligned}$$

Q. Given that  $[\vec{a}\vec{b}\vec{c}] = 5$

$$\begin{aligned} \therefore & [3(\vec{a} + \vec{b})\vec{b} + \vec{c} \ 2(\vec{c} + \vec{a})] \\ &= 6 [\vec{a} + \vec{b} \ \vec{b} + \vec{c} \ \vec{c} + \vec{a}] \\ &= 6 \times 2 [\vec{a} \ \vec{b} \ \vec{c}] = 6 \times 2 \times 5 = 60 \\ \therefore & (Q) \rightarrow (4) \end{aligned}$$

R. Given that  $\frac{1}{2}|\vec{a} \times \vec{b}| = 20 \Rightarrow |\vec{a} \times \vec{b}| = 40$

$$\begin{aligned} \therefore & \frac{1}{2} |(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})| = \frac{1}{2} | -2\vec{a} \times \vec{b} + 3\vec{b} \times \vec{a} | \\ &= \frac{1}{2} \times 5 |\vec{a} \times \vec{b}| = \frac{5}{2} \times 40 = 100 \\ \therefore & (R) \rightarrow (1) \end{aligned}$$

S. Given that  $|\vec{a} \times \vec{b}| = 30$

$$\begin{aligned} \therefore & |(\vec{a} + \vec{b}) \times \vec{a}| = |(\vec{b} \times \vec{a})| = 30 \\ \therefore & (s) \rightarrow (2) \end{aligned}$$

16.

(b) (P) - (4), (Q) - (3), (R) - (2), (S) - (1)

**Explanation:**

$$\begin{aligned} \text{P. } & \left[ \frac{1}{y^2} \left( \frac{\cos(\tan^{-1} y) + y \sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)} \right)^2 + y^4 \right]^{\frac{1}{2}} \\ &= \left[ \frac{1}{y^2} \left( \frac{\cos\left(\cos^{-1} \frac{1}{\sqrt{1+y^2}}\right) + y \sin\left(\sin^{-1} \frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1} \frac{\sqrt{1-y^2}}{y}\right) + \tan\left(\tan^{-1} \frac{y}{\sqrt{1-y^2}}\right)} \right)^2 + y^4 \right]^{\frac{1}{2}} \\ &= \left[ \frac{1}{y^2} \left( \frac{\frac{\sqrt{1+y^2}}{1}}{y(\sqrt{1-y^2})} \right)^2 + y^4 \right]^{\frac{1}{2}} \\ &= (1 - y^4 + y^4)^{\frac{1}{2}} = 1 \therefore (P) \rightarrow (4) \end{aligned}$$

Q.  $\cos x + \cos y = -\cos z \dots(i)$

and  $\sin x + \sin y = -\sin z \dots(ii)$

On squaring (i) and (ii) and then adding, we get

$$(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = \cos^2 z + \sin^2 z$$

$$\Rightarrow 2 + 2 \cos(x - y) = 1$$

$$\Rightarrow 4\cos^2 \frac{x-y}{2} = 1 \Rightarrow \cos \frac{x-y}{2} = \pm \frac{1}{2}$$

$$\therefore Q \rightarrow (3)$$

R.  $\cos\left(\frac{\pi}{4} - x\right) \cos 2x + \sin x \sin 2x \sec x$

$$= \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right) \cos 2x$$

$$\Rightarrow \cos 2x \left[ \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{\pi}{4} + x\right) \right]$$

$$= \sin 2x \sec x (\cos x - \sin x)$$

$$\Rightarrow 2 \sin \frac{\pi}{4} \sin x \cos 2x = 2 \sin x (\cos x - \sin x)$$

$$\Rightarrow 2 \sin x \left[ \frac{1}{\sqrt{2}} (\cos^2 x - \sin^2 x) - (\cos x - \sin x) \right] = 0$$

$$\Rightarrow 2 \sin x (\cos x - \sin x) \left( \frac{\cos x + \sin x}{\sqrt{2}} - 1 \right) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \tan x = 1 \text{ or } \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\Rightarrow x = 0 \text{ or } \frac{\pi}{4} \Rightarrow \sec x = 1 \text{ or } \sqrt{2}$$

$$\therefore (R) \rightarrow (2, 4)$$

S.  $\cot(\sin^{-1} \sqrt{1-x^2}) = \sin(\tan^{-1} x\sqrt{6})$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{1+6x^2}} \Rightarrow x = \pm \frac{1}{2} \sqrt{\frac{5}{3}}$$

$$\therefore (S) \rightarrow (1)$$

Hence (P)  $\rightarrow$  (4), (Q)  $\rightarrow$  (3), (R)  $\rightarrow$  (2, 4), (S)  $\rightarrow$  (1)

17. (a) P  $\rightarrow$  4; Q  $\rightarrow$  6; R  $\rightarrow$  5; S  $\rightarrow$  2

**Explanation:** Given 6 boys  $M_1, M_2, M_3, M_4, M_5, M_6$  and 5 girls  $G_1, G_2, G_3, G_4, G_5$ .

a.  $\alpha_1 \rightarrow$  Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls

$$\text{i.e., } {}^6C_3 \times {}^5C_2 = 20 \times 10 = 200 \therefore \alpha_1 = 200$$

b.  $\alpha_2 \rightarrow$  Total number of ways selecting at least 2 member and having equal number of boys and girls

$$\text{i.e., } {}^6C_1 {}^5C_1 + {}^6C_2 {}^5C_2 + {}^6C_3 {}^5C_3 + {}^6C_4 {}^5C_4 + {}^6C_5 {}^5C_5$$

$$= 30 + 150 + 200 + 75 + 6 = 461 \Rightarrow \alpha_2 = 461$$

c.  $\alpha_3 \rightarrow$  Total number of ways of selecting 5 members in which at least 2 of them girls

$$\text{i.e., } {}^5C_2 {}^6C_3 + {}^5C_3 {}^6C_2 + {}^5C_4 {}^6C_1 + {}^5C_5 {}^6C_0$$

$$= 200 + 150 + 30 + 1 = 381 \Rightarrow \alpha_3 = 381$$

d.  $\alpha_4 \rightarrow$  Total number of ways for selecting 4 members in which at least two girls such that  $M_1$  and  $G_1$  are not included together.

$$G_1 \text{ is included} \rightarrow {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$$

$$= 40 + 30 + 4 = 74$$

$$M_1 \text{ is included} \rightarrow {}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 30 + 4 = 34$$

$G_1$  and  $M_1$  both are not included

$${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$$

$$1 + 20 + 60 = 81$$

$$\therefore \text{Total number} = 74 + 34 + 81 = 189$$

$$\alpha_4 = 189$$

Now, P  $\rightarrow$  4; Q  $\rightarrow$  6; R  $\rightarrow$  5; S  $\rightarrow$  2

## Physics (MRQ)

18. (a) radiated power entering into one eye of the observer is in the range  $3.15 \times 10^{-8} \text{ W}$  to  $3.25 \times 10^{-8} \text{ W}$
- (b) the wavelength corresponding to the maximum intensity of light is 1160 nm
- (d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range  $2.75 \times 10^{11}$  to  $2.85 \times 10^{11}$

**Explanation:** According to question,

surface area of filament of light bulb,  $A = 64 \text{ mm}^2$

Temperature of filament,  $T = 2500 \text{ K}$

distance of bulb or source from observer,  $d = 100 \text{ m}$

Radius of the pupil of the eyes of the observer,  $R_e = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

a. Power radiated by the filament  $P = \sigma A e T^4$

$$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4 = 141.75 \text{ W} \quad (\because e = 1 \text{ for black body})$$

Hence, option (power radiated by the filament is in the range 642 W to 645 W) is incorrect.

b. Radiated power entering into one eye of the observer,

$$I = \frac{P}{4\pi d^2} \times (\pi R_1^2)$$

$$= \frac{141.75}{4\pi \times (100)^2} \times \pi \times (3 \times 10^{-3})^2 = 3.189375 \times 10^{-8} \text{ W}$$

Hence, option (radiated power entering into one eye of the observer is in the range  $3.15 \times 10^{-8} \text{ W}$  to  $3.25 \times 10^{-8} \text{ W}$ ) is correct

c. From wein's displacement law,  $\lambda_m T = b$

$$\text{or, } \lambda_m \times 2500 = 2.9 \times 10^{-3}$$

$$\text{or, } \lambda_m = 1.16 \times 10^{-6} = 1160 \text{ nm}$$

Hence, option (The wavelength corresponding to the maximum intensity of light is 1160 nm) is correct

d. Total no. of photons entering per second into one eye of the observer  $= \left(\frac{hc}{\lambda}\right) \times N_{\text{photons}} = I$

$$3.189375 \times 10^{-8} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1740 \times 10^{-9}} \times N_{\text{photons}}$$

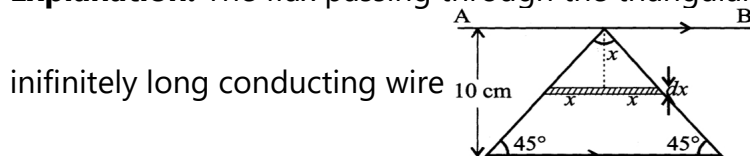
$$\therefore N_{\text{photons}} = 2.79 \times 10^{11}$$

Hence, option (taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range  $2.75 \times 10^{11}$  to  $2.85 \times 10^{11}$ ) is correct.

19. (b) There is a repulsive force between the wire and the loop

(d) The magnitude of induced emf in the wire is  $\left(\frac{\mu_0}{\pi}\right)$  volt

**Explanation:** The flux passing through the triangular wire if  $i$  current flows through the



$$d\phi = \int_0^{0.1} \frac{\mu_0 i}{2\pi x} \times 2\pi dx$$

$$\phi = \frac{\mu_0 i}{10\pi} = Mi$$

$$\therefore M = \frac{\mu_0}{10\pi} \left(\frac{dI}{dt} = 10 \text{ AS}^{-1} \text{ given}\right)$$

$$\text{Induced emf in the wire, } e = M \frac{di}{dt} = \frac{\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi} V$$

There will be no extra induced emf in the wire because there is no change in the magnetic flux due to rotation of loop.

As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

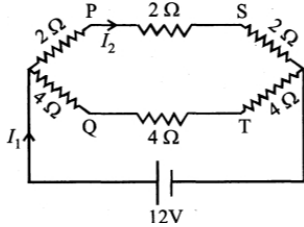
20. (a)  $I_2 = 2 \text{ A}$

(b) The current through PQ is zero

(c) The potential at S is less than that at Q.

(d)  $I_1 = 3 \text{ A}$

**Explanation:** Resistance of arm PQ and ST becomes ineffective as P & Q and S & T are at the same potential. The equivalent circuit is as shown in the figure.



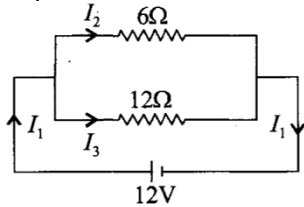
The resistance of the upper arm

$$R_1 = 2 \Omega + 2 \Omega + 2 \Omega = 6 \Omega$$

The resistance of the lower arm

$$R_2 = 4 \Omega + 4 \Omega + 4 \Omega = 12 \Omega$$

Equivalent resistance of the circuit,



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(6\Omega)(12\Omega)}{6\Omega + 12\Omega} = 4\Omega$$

$$\therefore I_1 = \frac{12V}{4\Omega} = 3 \text{ A} \quad I_2 = \left( \frac{12}{6+12} \right) \times 3 = 2 \text{ A}$$

$$I_3 = I_1 - I_2 = 1 \text{ A}$$

Potential difference across A and P,

$$V_A - V_P = I_2 \times 2 \Omega = (2 \text{ A})(2 \Omega)$$

$$12V - V_P = 4V \text{ or } V_P = 8V$$

Potential difference across A and Q

$$V_A - V_Q = I_3 \times 2 \Omega = (1 \text{ A})(4 \Omega)$$

$$12V - V_Q = 4V$$

$$V_Q = 12V - 4V = 8V$$

Potential difference across P and S,

$$V_P - V_S = (2 \text{ A})(2 \Omega) = 4V$$

$$8V - V_S = 4V \Rightarrow V_S = 4V$$

$$V_S < V_Q$$

### Physics (MCQ)

21.

(d) short wavelength

**Explanation:** short wavelength

22.

$$(d) \frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{\frac{1}{2}}$$

**Explanation:** The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular directions.

$$\therefore B = \sqrt{B_1^2 + B_2^2}$$

$$\therefore B = \frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$$

23.

$$(c) \sqrt{920 \times 980} \text{ cm/sec}$$

**Explanation:** Let  $d_w$  and  $d_o$  be the densities of water and oil; then the pressure at the bottom of the tank

$$= h_w d_w g + h_o d_o g$$

Let this pressure be equivalent to pressure due to water of height  $h$ .

$$\text{Then, } h d_w g = h_w d_w g + h_o d_o g$$

$$\therefore h = h_w + \frac{h_o d_o}{d_w} = 100 + \frac{400 \times 0.9}{1}$$

$$= 100 + 360 = 460$$

According to Toricelli's theorem,

$$v = \sqrt{2gh} = \sqrt{2 \times 980 \times 460} \text{ cm/sec}$$

$$= \sqrt{920 \times 980} \text{ cm/sec}$$

24.

$$(c) n = 5, f_2 = \left(\frac{5}{4}\right) f_1$$

**Explanation:**  $f_1 = \frac{v}{l}$  (2nd harmonic of open pipe)

$$f_2 = n \left(\frac{v}{4l}\right) \text{ (nth harmonic of closed pipe)}$$

Here,  $n$  is odd and  $f_2 > f_1$ . It is possible when  $n = 5$

because with  $n = 5$

$$f_2 = \frac{5}{4} \left(\frac{v}{l}\right) = \frac{5}{4} f_1$$

### Physics (NUM)

25. 5

Explanation:

As two successive harmonics are found to occur at frequency 750 Hz and 1000 Hz and frequency

$$f = \frac{P}{2l} \sqrt{\frac{T}{\mu}} \text{ So,}$$

$$750 = \frac{P}{2} \sqrt{\frac{T}{\mu}} \dots (i)$$

$$\text{and, } 1000 = \frac{P+1}{2} \sqrt{\frac{T}{\mu}} \dots (ii)$$

Dividing eq (ii) by (i)

$$\frac{4}{3} = \frac{P+1}{P} \therefore P = 3$$

Putting this value of  $P = 3$  in eq. (ii) and solving we get tension  $T$

$$1000 = \frac{4}{2} \sqrt{\frac{T}{2 \times 10^{-5}}} \therefore T = 5 \text{ N}$$

26. 0.95

Explanation:

After entering in new region, time taken by projectile to reach ground is given as

$$t = \sqrt{\frac{2h}{g_{eff}}} = \sqrt{\frac{2 \times 0.81 \times u^2 \sin^2 \theta}{g \times 2g}} = 0.9 \frac{u \sin \theta}{g}$$



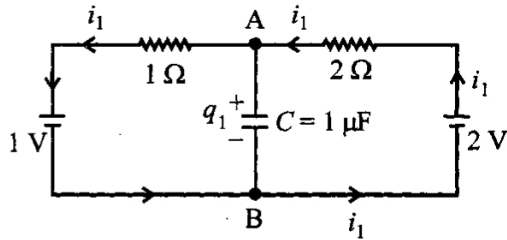
So, horizontal displacement done by projectile in new region is given as

$$x = 0.9 \times \frac{u \sin \theta}{g} \times u \cos \theta = 0.9 \left( \frac{d}{2} \right)$$

$$\text{Now, new range} = \frac{d}{2} + 0.9 \frac{d}{2} = 0.95d$$

27. 1.33

Explanation:



When switch is connected to position P From KVL,

$$V_A - 1 \cdot i_1 - 1 + 2 - 2i_1 = V_A \Rightarrow 3i_1 = 1 \therefore i_1 = \frac{1}{3} \text{ A}$$

$$\text{Again } V_A - 1 \cdot i_1 - 1 = V_B \text{ or, } V_A - V_B = 1 + i_1 = \frac{4}{3} \text{ V}$$

$$\text{Potential drop across capacitor } \Delta V = \frac{4}{3} \text{ V}$$

$$\therefore \text{Charge on capacitor, } q_1 = C \Delta V = 1 \times \frac{4}{3} \mu\text{C}$$

$$q_1 = 1.33 \mu\text{C}$$

28. 8

Explanation:

$$\text{We know that } N = N_0 e^{-\lambda t}$$

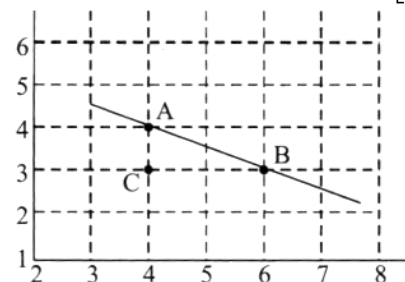
$$\therefore \frac{dN}{dt} = N_0 e^{-\lambda t} (-\lambda) = -\lambda N$$

Taking log on both sides

$$\log_e \frac{dN}{dt} = \log_e (\lambda N_0) - \lambda t$$

Comparing it with the graph line,

$$\text{Decay constant, } \lambda = \frac{1}{2} yr^{-1} \left[ \frac{AC}{BC} = \frac{1}{2} \right]$$



$$\therefore T_{1/2} = \frac{0.693}{\lambda} = 0.693 \times 2 = 1.386 \text{ years}$$

$$n(t_{1/2}) = 4.16 \therefore n = \frac{4.16}{1.386} \approx 3$$

$$\therefore N = N_0 \left( \frac{1}{2} \right)^3$$

$$\therefore \frac{1}{P} = \frac{1}{8} [\therefore P = 8]$$

29. 24.0

Explanation:

$$\text{Given, intensity of light, } I = 256 \frac{\text{W}}{\text{m}^2}$$

By first polaroid P<sub>1</sub> intensity will be halved.

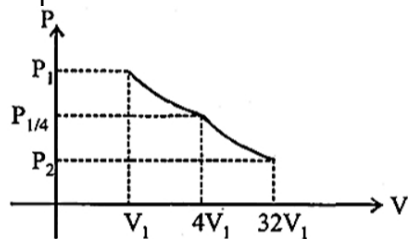
$$\text{After P}_2, I_2 = \frac{I}{2} \cos^2 60^\circ = \frac{I}{8}$$

$$\text{After P}_3, I_3 = \frac{I}{8} \cos^2 30^\circ = \frac{I}{8} \left( \frac{\sqrt{3}}{2} \right)^2 = \frac{3I}{32}$$

$$\text{Intensity} = \frac{256 \times 3}{32} = 24 \frac{\text{W}}{\text{m}^2}$$

30. 1.78

Explanation:



For monatomic gas,  $\gamma = \frac{5}{3}$

In adiabatic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow \frac{P_1}{4} (4V_1)^{5/3} = P_2 (32V_1)^{5/3}$$

$$\Rightarrow P_2 = \frac{P_1}{4} \left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$$

$$\text{And } W_{\text{adi}} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{P_1 V_1 - \frac{P_1}{128} (32 V_1)}{\frac{5}{2} - 1}$$

$$= \frac{P_1 V_1 (3/4)}{2/3} = \frac{9}{8} P_1 V_1$$

In isothermal process,

$$W_{\text{iso}} = 2.303 \mu R T \log_{10} \left( \frac{V_2}{V_1} \right)$$

$$\Rightarrow W_{\text{iso}} = P_1 V_1 \ln \left( \frac{4 V_1}{V_1} \right) = 2 P_1 V_1 \ln 2$$

$$\therefore \frac{W_{\text{iso}}}{W_{\text{adia}}} = \frac{2 P_1 V_1 \ln 2}{\frac{9}{8} P_1 V_1} = \frac{16}{9} \ln 2 = f \ln 2$$

$$\text{So, } f = \frac{16}{9} = 1.7778 \approx 1.78$$

### Physics (MATCH)

31.

(c) P  $\rightarrow$  3; Q  $\rightarrow$  2; R  $\rightarrow$  4; S  $\rightarrow$  1

**Explanation:**  $\therefore$  Orbital velocity,

$$V = \sqrt{\frac{GM}{R}}, \text{ or, } V \propto \frac{1}{\sqrt{R}} \therefore \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_1 v_2 R_2} = \frac{2 \times 2 \times 1}{1 \times 1 \times 4} = \frac{1}{1}$$

$$\text{Kinetic energy, } K = \frac{GMm}{2R}$$

$$\therefore \frac{k_1}{k_2} = \frac{m_1}{m_2} \times \frac{R_1}{R_2} = \frac{2 \times 4}{1 \times 1} = \frac{8}{1}$$

From Kepler's law of planetary motion.

$$T^2 \propto R^3 \therefore \frac{T_1}{T_2} = \left( \frac{R_1}{R_2} \right)^{\frac{3}{2}} = \frac{1}{8}$$

32.

(c) P  $\rightarrow$  3, Q  $\rightarrow$  1, R  $\rightarrow$  4, S  $\rightarrow$  2

**Explanation:** As,  $V = V_0 \sin \omega t$  and resonant angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = 10^5 \text{ rad/s} = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2}$$

$$= \frac{1}{5 \times 10^{-2} \times 10^{10}} = 2 \times 10^{-9} \text{ F}$$

$$[\therefore L = 50 \text{ mH} = 5 \times 10^{-2} \text{ henry}]$$

$$I_0 = \frac{V_0}{R} = \frac{45}{R} [\therefore V = 45 \sin \omega t \text{ given}]$$

$$\omega = 8 \times 10^4 \text{ rad/s} = 0.8 \omega_0$$

$$I = 0.05 I_0 = \frac{I_0}{20} \Rightarrow Z = 20R$$

$$X_L = L\omega = 8 \times 10^4 \times 5 \times 10^{-2} \Omega = 4k\Omega$$

$$X_C = \frac{1}{C\omega} = \frac{1}{8 \times 10^4 \times 2 \times 10^{-9}} = \frac{1}{16} \times 10^5 \Omega = \frac{25}{4} k\Omega$$

$$Z^2 = R^2 + (X_C - X_L)^2 \text{ or, } 400 R^2 = R^2 + \left(\frac{9}{4} k\Omega\right)^2$$

$$\therefore R = \frac{\frac{9}{4} k\Omega}{\sqrt{399}} = \frac{9}{80} k\Omega = \frac{900}{8} \Omega$$

$$I_0 = \frac{V_0}{R} = \frac{45 \times 8}{900} = \frac{8}{20} \text{ A} \approx 0.4 \text{ A} = 400 \text{ mA}$$

Quality factor,

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{8}{900} \sqrt{\frac{5 \times 10^{-2}}{2 \times 10^{-9}}} = \frac{8}{900} \sqrt{25 \times 10^6}$$

$$= \frac{8}{900} \times 5000 = 44.4$$

$$Q = \frac{\omega_0}{\Delta\omega} \therefore \text{Bandwidth, } \Delta\omega = \frac{\omega_0}{Q} = \frac{10^5}{44.4} = 2250.0$$

$$\text{Peak Power dissipated, } P_{\max} = I_0^2 R = \frac{45^2}{R^2} \times R = \frac{45^2}{R}$$

$$= \frac{45^2}{900} \times 8 = 18.4 \text{ W} \approx 18 \text{ W}$$

33. (a) P → 3, Q → 4, R → 2, S → 1

**Explanation:** From Wien's displacement law,  $\lambda_m T = b$  (constant) and  $b = 2.9 \times 10^{-3} \text{ mK}$

$$\text{When } T = 200 \text{ K (P), } \lambda_m = \frac{b}{T} = \frac{2.9 \times 10^{-3}}{2000}$$

$$= 1450 \text{ nm (max)}$$

When  $T = 3000 \text{ K (Q)}$

$$\lambda_m = \frac{2.9 \times 10^{-3}}{3000} = 966.66 \text{ nm}$$

When  $T = 5000 \text{ K (R)}$

$$\lambda_m = \frac{2.9 \times 10^{-3}}{5000} = 580 \text{ nm}$$

When  $T = 10000 \text{ K (S)}$

$$\lambda_m = \frac{2.9 \times 10^{-3}}{10000} = 290 \text{ nm min}$$

For option (P)  $\lambda$  maximum and  $\beta = \frac{\lambda D}{d}$

$\therefore$  widest central maximum

For option (Q) Power

$$P_{3000} = \sigma A (3000)^4$$

$$P_{6000} = \sigma A (6000)^4$$

$$\frac{P_{3000}}{P_{6000}} \left(\frac{1}{2}\right)^4 = \frac{1}{16} \therefore P_{3000} = \frac{1}{16} P_{6000}$$

For (R) Wavelength  $\lambda = 580 \text{ nm}$

Visible to human eyes

$$\text{For (S) } \lambda = \frac{hc}{\phi} = \frac{1.24 \times 10^{-6}}{4} = 30 \text{ nm}$$

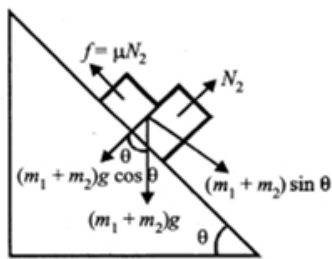
$290 \text{ nm} < 310 \text{ nm}$  so this radiation lead to emission of photo electrons from a metal of work function  $\phi = 4 \text{ eV}$ .

For imaging human bones x-rays wavelength range  $1 - 10 \text{ nm}$  is used.

34.

(c) (P) - (i), (Q) - (ii), (R) - (ii), (S) - (iii)

**Explanation:** Block will not slip or will be at rest if



$$(m_1 + m_2) g \sin \theta \leq \mu m_2 g \cos \theta$$

$$\tan \theta \leq \frac{\mu m_2 g}{(m_1 + m_2) g}$$

$$\Rightarrow \tan \theta \leq \frac{\mu m_2}{m_1 + m_2}$$

$$\Rightarrow \tan \theta \leq \frac{0.3 \times 2}{1 + 2} \leq \frac{1}{5}$$

$$\Rightarrow \tan \theta \leq 0.2 \text{ i.e., } \theta \leq 11.5^\circ$$

i.e., If the angle  $0 < 11.5^\circ$  the frictional force is less than

$$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6 g$$

and is equal to  $(m_1 + m_2) g \sin \theta$

Blocks will not slip on the inclined plane and friction is static.

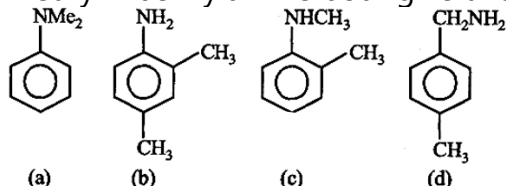
At  $\theta > 11.5^\circ$  the bodies start moving on the inclined plane and friction is kinetic and equal to  $\mu m_2 g \cos \theta$

### Chemistry (MRQ)

35. (b) 2, 4-dimethylaniline

(c) p-methylbenzylamine

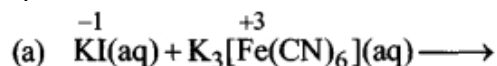
**Explanation:** Only primary amines give carbylamine test. Hence 2, 4-dimethylaniline and P-methyl—benzylamine both give this test.



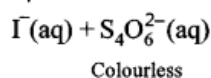
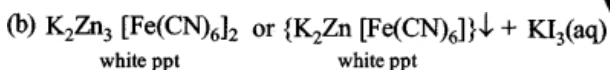
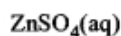
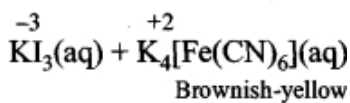
36. (a) The first reaction is a redox reaction.

(c) Addition of filtrate to starch solution gives blue colour.

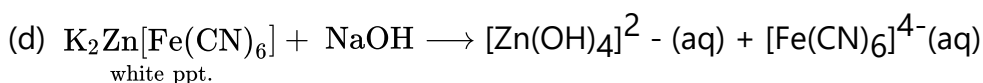
(d) White precipitate is soluble in NaOH solution.

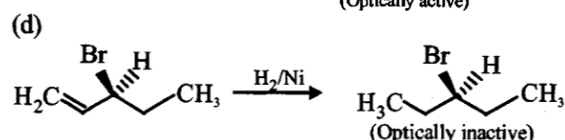
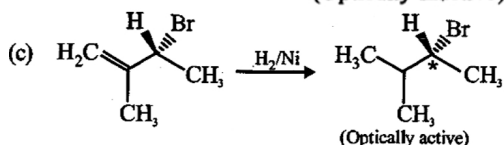
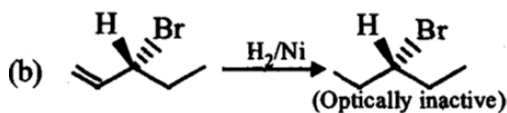
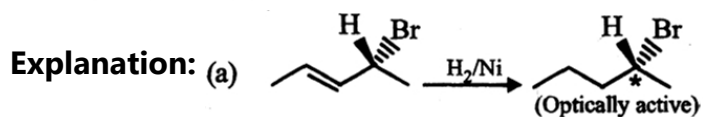
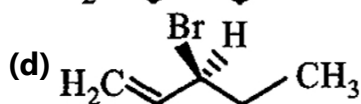
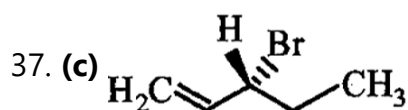


**Explanation:**



(c) When the filtrate containing  $\text{KI}_3$  is added to starch solution, the dissolved  $\text{I}_2$  will produce a blue colour solution.





### Chemistry (MCQ)

38. (a) Nitric acid

**Explanation:** For the test of halogens, it is necessary to remove NaCN and Na<sub>2</sub>S from the sodium extract if nitrogen and sulphur are present. This is done by boiling the sodium extract with conc. HNO<sub>3</sub>. If NaCN and Na<sub>2</sub>S are not decomposed, white or black ppt of AgCN and Ag<sub>2</sub>S respectively are formed with AgNO<sub>3</sub> solution.

39.

(c) Y > Z > X

**Explanation:** Lower the value of E<sup>o</sup>, stronger the reducing agent. Reducing power:

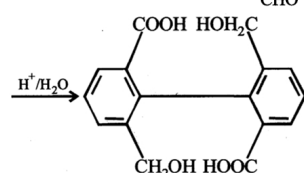
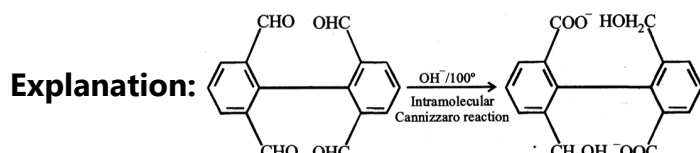
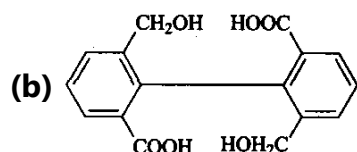
Y (E<sup>o</sup> = -3.03 V) > Z (E<sup>o</sup> = -1.18 V) > X (E<sup>o</sup> = 0.52 V)

40.

(b) NH<sub>3</sub>

**Explanation:** Amongst XH<sub>3</sub> where 'X' is group-15 elements, basic strength decreases from top to bottom. Hence, NH<sub>3</sub> is the strongest base.

41.



The above reaction is an example of intramolecular Cannizzaro reaction.

### Chemistry (NUM)

42. 6

Explanation:

The formula for conductance is  $G = \kappa \times \frac{a}{l}$

$$5 \times 10^{-7} = \kappa \times \frac{1}{120}$$

$$\kappa = 6 \times 10^{-5} \text{ S cm}^{-1}$$

$$\Lambda_m^c = \frac{\kappa \times 1000}{M} = \frac{6 \times 10^{-5} \times 1000}{0.0015} = 40$$

$$\therefore \text{pH} = 4$$

$$\therefore [\text{H}^+] = 10^{-4} = c\alpha = 0.0015 \alpha$$

$$\alpha = \frac{10^{-4}}{0.0015}$$

$$\text{Also, } \alpha = \frac{\Lambda_m^c}{\Lambda_m^o} \Rightarrow \frac{10^{-4}}{0.0015} = \frac{40}{\Lambda_m^o}$$

$$\Lambda_m^o = \frac{40 \times 0.0015}{10^{-4}} = 6 \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$$

Hence,  $Z \approx 6$

43. 104

Explanation:

Given arrangement represents octahedral void and for this

$$\frac{r_{+}(\text{cation})}{r_{-}(\text{anion})} = 0.414$$

$$\frac{r(\text{A}^+)}{r(\text{X}^-)} = 0.414$$

$$= r(\text{A}^+) = 0.414 \times r(\text{X}^-) = 0.414 \times 250 \text{ pm}$$

$$= 103.5 \text{ pm} \approx 104 \text{ pm}$$

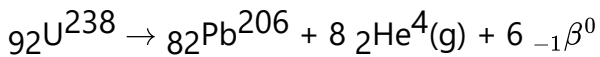
44. 3

Explanation:

KCN, K<sub>2</sub>CO<sub>3</sub> and LiCN are the salts of weak acid and strong base. So, their aqueous solutions turn red litmus paper blue.

45. 9

Explanation:



$$n(\text{gas})[\text{Initial}] = 1 (\text{air})$$

$$n(\text{gas})[\text{Final}] = 8 (\text{He}) + 1 (\text{air}) = 9$$

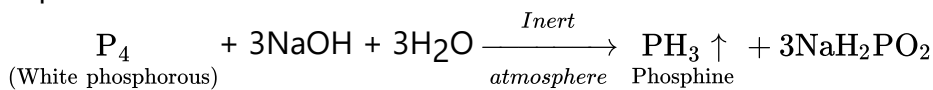
$\Rightarrow$  At constant temperature and volume;

$$p \propto n$$

$$\text{So, } \frac{p_f}{p_i} = \frac{n_f}{n_i} = \frac{9}{1} = 9$$

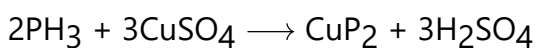
46. 2.38

Explanation:



PH<sub>3</sub>: a non-inflammable gas in its pure form; slightly soluble in water.

When PH<sub>3</sub> is absorbed in CuSO<sub>4</sub> solution cupric phosphide forms:



$$1 \text{ mol of P}_4 = 31 \times 4 = 124 \text{ g}$$

$$\therefore 1.24 \text{ g of white phosphorous} = 0.01 \text{ mol}$$

$$\therefore 0.01 \text{ mol of P}_4 \text{ forms } 0.01 \text{ mol of P}_3$$

$$\text{No. of moles of CuSO}_4 \text{ is required for complete consumption of } 0.01 \text{ mol} = 0.01 \times \frac{3}{2} = 15 \times 10^{-3}$$

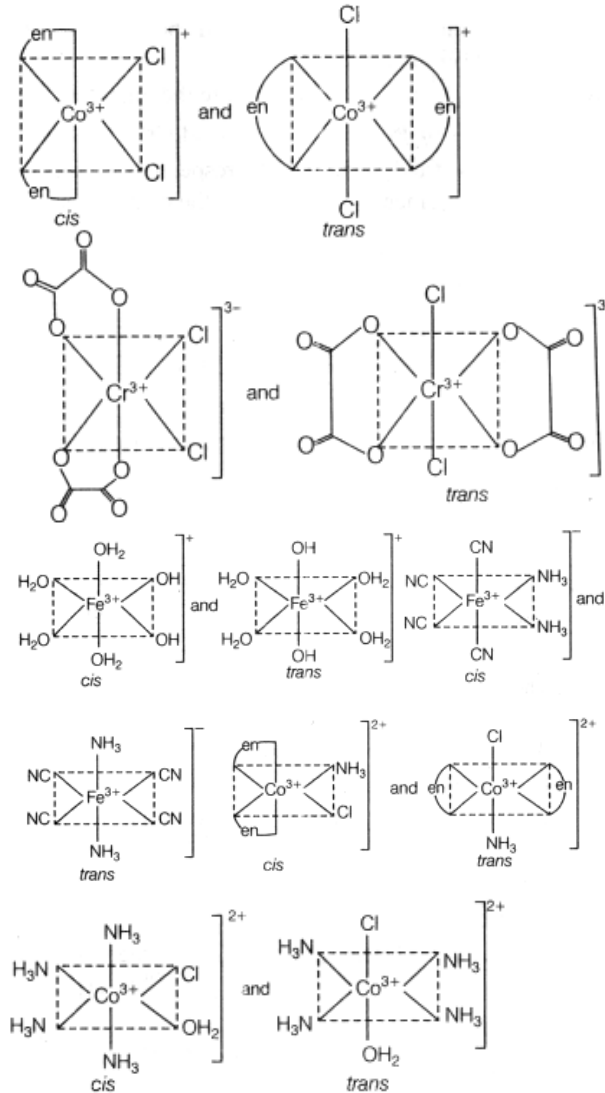
M. W. of  $\text{CuSO}_4 = 159 \text{ g/mol}$

$\therefore$  Amount of  $\text{CuSO}_4$  required =  $15 \times 10^{-3} \times 159 = 2.38 \text{ g}$

47.6

Explanation:

All six complex will show cis-trans isomerism.

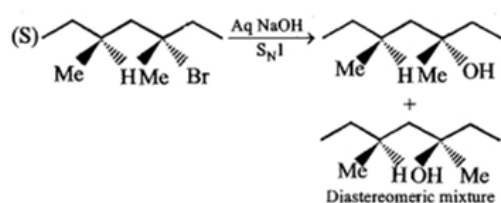
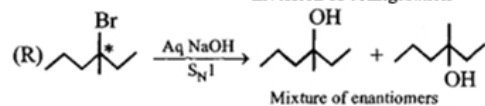
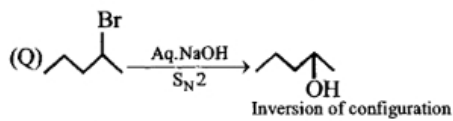
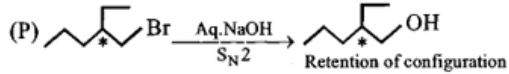


### Chemistry (MATCH)

48.

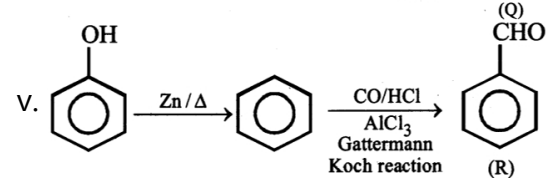
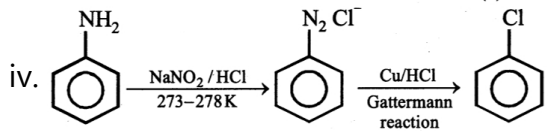
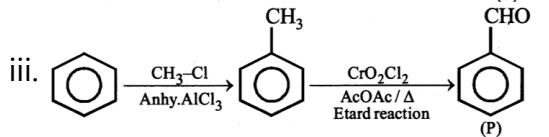
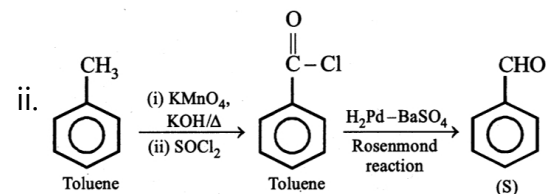
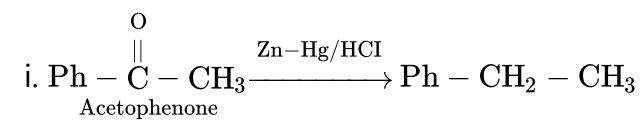
(b)  $\text{P} \rightarrow 2$ ;  $\text{Q} \rightarrow 1$ ;  $\text{R} \rightarrow 3$ ;  $\text{S} \rightarrow 5$

**Explanation:**  $\text{P} \rightarrow 2$ ,  $\text{Q} \rightarrow 1$ ,  $\text{R} \rightarrow 3$ ,  $\text{S} \rightarrow 5$



49. (a) P → 3; Q → 4; R → 5; S → 2

**Explanation:** P → 3, Q → 4, R → 5, S → 2



50.

(b) P - 1; Q - 5; R - 4; S - 1

**Explanation:** P - 1; Q - 5; R - 4; S - 1

51.

(b) (I), (T)

**Explanation:**  $r \propto \frac{n^2}{Z}$  or  $r = 0.529 \times \frac{n^2}{Z}$ ; (I), (T)

$|L| \propto n$  or  $mvr = \frac{nh}{2\pi}$ ; (II), (S)